Detecting Emergent Behaviors with Semi-Boolean Algebra

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As aerospace and military systems become increasingly interconnected, their combined behavior can become more complex than each individual subsystem in isolation and unexpected self-organizing or emergent behaviors can result. Explicit consideration of emergent behavior has not been addressed in current systems development methodologies. This paper gives background on approaches for modeling emergent behavior and discusses the use of semi-Boolean algebra as a means for detecting emergence in combined behaviors. Semi-Boolean algebra is a generalization of the Boolean algebra by weakening the requirement that any two elements have a common upper bound. The paper also shares an example approach for modeling emergent behavior and describes several ways to detect emergent behavior with this technique.

I. Introduction

INTERCONNECTED systems are becoming standard in aerospace, military, and commercial applications. As systems are interconnected, their combined behavior is far more complex than each individual system in isolation. Unexpected self-organizing or emergent behaviors can result from the combination of state spaces of the systems[1,2]. Bonabeau and others who studied self-organization in social insects, stated, "complex collective behaviors may emerge from interactions among individuals that exhibit simple behaviors." He described emergent behavior as, "a set of dynamical mechanisms whereby structures appear at the global level of a system from interactions among its lower-level components[3,4,5]." Heylighen stated that, "complex systems consist of many parts that are connected through their interaction." Heylighen goes on to say that, "their components are both distinct and connected, both autonomous and to some degree mutually dependent[6]." The idea of emergence is well known from biology, economics, and other scientific areas, but the concept is not well understood by computer scientists and engineers.

The emergent properties of systems of systems make them powerful, but at the same time make them more difficult to model, design, and verify. Learning and adaptation within these systems cause additional concerns because emergent behavior simply cannot be fully considered through the use of traditional system development methods, such as testing and model checking. In addition, self-organization can occur as the individual systems optimize to address inefficiencies in the larger system. Though the systems may be optimizing at the individual system level, the larger system may be sub-optimized as a result, and cause inefficiencies, new errors, unexplained behaviors and new security openings not previously available.

The remainder of this paper discusses emergent behavior in general and then discusses a mathematical technique called semi-Boolean algebra that can be used to identify that emergent behavior has occurred, or what it would take for it to occur. This technique is then described in its application to detecting emergent behavior in a multi-sensor system.

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II. Emergent Behavior

The use of emergent behavior, particularly through swarm-based technologies, has been steadily increasing and making its way out of the laboratory into real systems. Modeling, design, and verification of these types of systems have not kept up with the deployment of the technology. In this section, we set the stage for introduction in section III of semi-Boolean algebra to model or detect the presence of emergent behavior in a multi-sensor system. This ability can then be used as a component of system of systems verification.

Bonabeau et al., who studied self-organization in social insects, stated "that complex collective behaviors may emerge from interactions among individuals that exhibit simple behaviors" and described emergent behavior as "a set of dynamical mechanisms whereby structures appear at the global level of a system from interactions among its lower-level components[3,4,5]." Heylighen states that "complex systems consist of many parts that are connected through their interaction[20]." Heylighen goes on to say that "their components are both distinct and connected, both autonomous and to some degree mutually dependent." Complex collective behaviors can form from swarms, systems of systems, social networks and other interconnected systems. The communication and interaction between individual systems that make up a larger system can produce some of the same collective or emergent behavior as has been seen in social insects and swarm-based systems. Davis described emergent behavior as "emergence can represent a valid behavior arising from seemingly unrelated phenomena, or it can reflect an error in a model or its implementation[41]. Johnson described behavior as being emergent if it is unexpected and stems from the interactions of the underlying components of the model[38].

Aggregate behavior is different from emergent behavior. Aggregate behavior is defined as formed by the collection of units or particles into a body, mass, or amount. So the aggregate behavior should be easily predictable based on the assembly of the parts, while emergent behavior provides a greater-than-the-sum type of behavior.

David J. Chalmers presented one of the earliest emergent behavior taxonomies, which distinguishes between *weak* and *strong* emergence[40]. Strong emergence is not deducible even in principle from the laws of the low-level domain, while weak emergence is only unexpected given the properties and principles of the low-level domain.

III. Semi-Boolean Algebra for Representing and Inferring Emergent Behavior

Behavior analysis of complex, interconnected systems is often complicated by the absence of a unifying, "god's eye" view of the overall system of systems. While a unifying analysis perspective may not be available, it is often the case that there are maximal, overlapping perspectives corresponding to the component systems or sub-networks. A mathematical structure that has similar characteristics is a *dominated semi-Boolean algebra*. (Strictly speaking, the semi-Boolean algebras described in this paper are the type also known as subtraction algebras[35]. A dual type of semi-Boolean algebra derived by weakening the requirements for common lower bounds can also be constructed. The two types of semi-Boolean algebras can be shown to be isomorphic.) Semi-Boolean algebra is a generalization of the Boolean algebra concept obtained by weakening the requirement that any two elements have a common upper bound. As a consequence, there is not a global sense of complementation. For example, consider a simple case where the system of systems involves two mobile sensors. One sensor is a simple acoustic sensor, with possible readings of "No signal," "Low signal," or "High signal." The set of readings is represented as **{N, LoAc, HiAc}**. The second sensor is similar but is an infrared sensor with analogous set of readings **{N, LoIR, HiIR}**. We identify the "No signal" readings from the two sensors as being equivalent. The lattice diagram of potential sensor events under the partial ordering derived from set inclusion is shown in Figure 1.

The presence of local maximal elements **1Ac** and **1IR** are referred to as *operations*[36]. The ideals generated by the maximal elements, i.e., the subsets of each of these elements, form a Boolean algebra, but the overall structure is not Boolean. In these ideals we can speak of relative complementation. The relationship of being relative complements is referred to as the *operational complement* (oc) relationship[37]. For example, considering the ideal generated by the element **1Ac**, we see that **{N}** *oc* **{HiAc, LoAc}**. That is, in terms of the operation **1Ac** that we can describe as "Use the acoustic sensor," the event {**N**} = "No detection occurred" is an operational complement to the event {**HiAc**, **LoAc**} = "Some acoustic detection occurred." Note also that considering the ideal generated by the operation "Use the IR sensor," **1IR,** we see that **{N}** *oc* **{HiIR, LoIR}**. When elements have a common operational complement, we say that they are *operationally perspective* (op). Hence, **{HiAc, LoAc}** *op* **{HiIR, LoIR}**. That is, the events "Some acoustic detection occurred" and "Some IR detection occurred" are operationally perspective via the common operational complement "No detection occurred". It is shown that the op relationship is an equivalence relationship when the semi-Boolean algebra satisfies the *Manual condition*, M.

(M) Given a dominated semi-Boolean algebra **A** with distinct elements **B**, **C**, **D**
where **B** *op* **C** and **C** *oc* **D** there exists a maximal element **E** in **A** such that **A**, **C** □ **E** and **A** *oc* **C**.

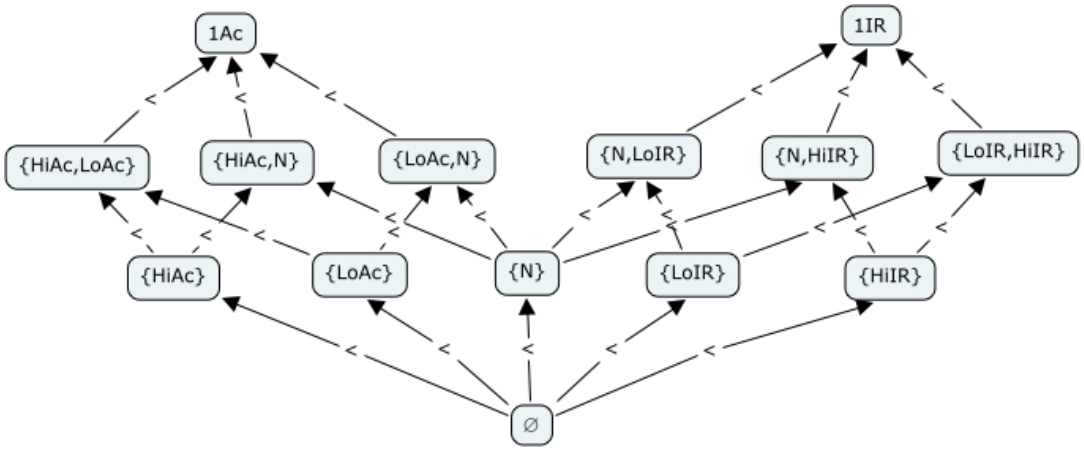

Figure 1. Dominated semi-Boolean algebra for the multiple sensor situation.

Once we have a semi-Boolean algebra satisfying condition M we can form a quotient structure modulo the *op* relationship. This structure is known as the *op-logic* for the semi-Boolean algebra. The op-logic admits a partial order induced by the partial order from the original semi-Boolean structure. The order diagram for the op-logic corresponding to our example is shown in Figure 2, with "interesting" equivalence classes (in the sense that they are fusions of elements from distinct operations) shown in green.

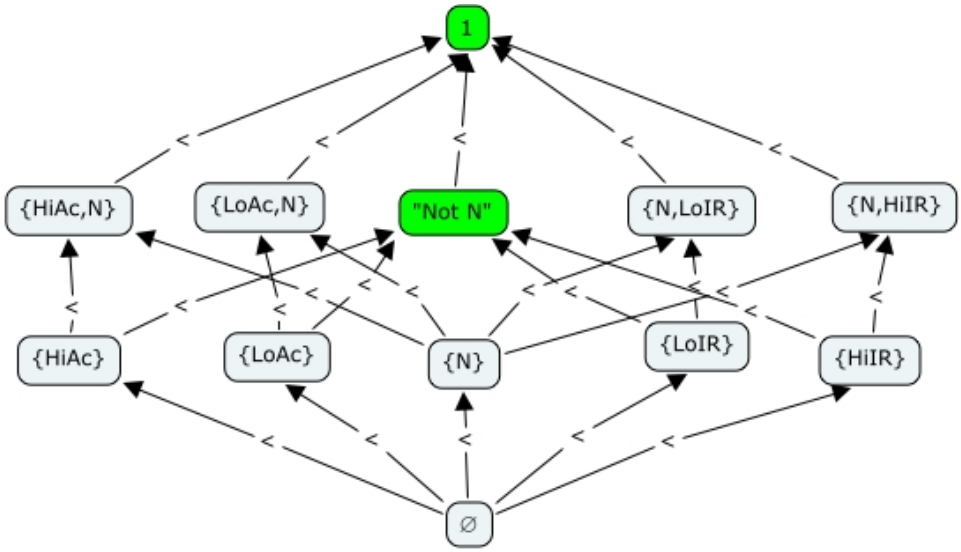

Figure 2. Op-logic for the multiple sensor situation.

The op-logic under the partial ordering relation is not, in general, Boolean, semi-Boolean, or even necessarily a lattice. However, op-logic does form a structure defined as an associative ortho-algebra[38]. This structure includes an incomplete ortho-complementarity relation derived from the *oc* relationship on the original semi-Boolean algebra. Considering our example, the elements {**HiAc**}, {**LoAc**}, {**LoIR**}, and {**HiIR**} are all less than the element annotated "**Not N**," which is the equivalence class for the events {**HiAc**, **LoAc**} and {**LoIR**, **HiIR**}. The elements "**N**" and "**Not N**" are orthocomplements, stemming from the event {**N**} being operationally complementary to the events in the equivalence class "**Not N**."

In our example, the op-logic is not Boolean or semi-Boolean, but is a lattice. However, certain regularity conditions in the overlap structure between the various operations the op-logic can show a structure supporting a greater degree of inference, such as an orthomodular lattice[36]. A future research goal is to characterize the real-world situations that produce these regularity conditions.

Given the semi-Boolean representational structure and resulting op-logic inference structure, we consider several ways to describe and discover emergent behaviors. The approach can be summarized:

- The portions of the network involved in the various emergent behaviors form a partially ordered set (poset) under a specialized inclusion relationship that can be approximated by the term "sub-network of." In our example, this is the combination of the response outcomes from the two sensors, shown in Figure 1. This poset structure has a set of maximal elements. The notion of operational complementation makes sense with respect to these maximal elements. However, there is no global complementation relation.
- The poset structure can be used to induce an inference structure, the op-logic, based on equivalence classes under relative complementation.
- Candidate emergent behaviors correspond to nodes in the inference structure that are equivalence classes containing members from different sub-networks. These are the "interesting" elements in the op-logic, like the element "**Not N**" in Figure 2.
- The inference structure obeys order axioms yet to be determined. While, in general, these axioms are weaker than those of Boolean algebra, they do provide some utility in reasoning across the entire family of overlapping networks and corresponding emergent behaviors. In our example, this includes inference of the "**Not N**" behavior from the detection of any of the atomic behaviors **HiAc**, **LoAc**, **LoIR**, **HiIR**.

The approach above provides the potential for qualitative discovery of explicit emergent behavior. It is also desirable to infer implicit emergent behavior. Typically, the component systems have multiple outputs and multiple overlapping state variables. Structural equation modeling (SEM) is one technique for modeling systems with latent, hidden state variables[39]. SEM supports estimating these hidden variables given the inputs and outputs of the system. Additionally, SEM supports identification and confirmation of causal hypotheses. Another future research goal is to investigate how simultaneous, overlapping structured equation models can be expressed on the semi-Boolean structure in order to derive estimates of the unobservable parameters that could govern the emergent behavior[34,35].

IV. Conclusion

Current and future aerospace systems will increasingly use system of systems as a basis for new systems. Due to the nature of these systems, they may unintentionally exhibit emergent behaviors. Consequently, these missions will pose an even greater challenge for modeling than in past missions. This paper gives an approach for discovering emergent behavior through the use of semi-Boolean Algebra. There were four possible ways to describe or infer emergent behavior: through operational complementation, inducing it through the poset structure, equivalent classes in the structure, and through the inference structure via order axioms. Future work in this area includes inferring implicit emergent behavior through techniques such as structural equation modeling.

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